P. MATHS.
w. MECHS. I

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UNIVERSITIES OF MANCHESTER LIVERPOOL
LEEDS SHEFFIELD AND BIRMINGHAM

JOINT MATRICULATION BOARD

GENERAL CERTIFICATE OF EDUCATION

PURE MATHEMATICS WITH MECHANICS (0 26), PAPER 1

ORDINARY

Wednesday 26 June 1963 9-30—12

Negligently presented or slovenly work will be penalized.

Mathematical tables (green covers) will be provided.

Answer **seven** *questions.*

1. Three motorists A, B and C start at the same time from a town X and travel with constant speeds V_1 m.p.h., V_2 m.p.h., V_3 m.p.h. respectively towards a town Y, distant a miles from X. A fourth motorist D starts at the same time from Y and travels with a constant speed V_2 m.p.h. towards X. Show that A and D will meet $a/(V_1 + V_2)$ hr. after starting.

Show that, if D meets A, B and C at times that are in arithmetic progression, then V_1, V_2 and V_3 will be in geometric progression.

Calculate the speeds of all four motorists if the distance between X and Y is 70 miles and if D meets A, B and C 50, 60 and 70 min. respectively after leaving Y.

[Turn over

2. (a) Show that the term independent of x in the

expansion of $\left(\frac{3}{x^3}-\frac{x}{9}\right)^{12}$ is $-\frac{220}{3^{15}}$

(*b)* Show that, if $z = y^5 - 4y^4 + 7y^3 - 6y^2 + 3y$ where $y = x+1$ and *x* is not equal to 1, then

$$z = \frac{x^6 - 1}{x-1}$$

3. Prove that, if $t = \tan\frac{\theta}{2}$ then

$$\frac{2t}{1+t^2} = \sin\theta \text{ and } \frac{1-t^2}{1+t^2} = \cos\theta$$

Show that the equation $a\cos\theta + b\sin\theta = c$ has two unequal roots between 0 and 2π if $c^2 < a^2 + b^2$

Show that in this case, if α and β are the roots of this equation, then

(i) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{c+a}$

(ii) $\tan\frac{1}{2}(\alpha+\beta) = \frac{b}{a}$

4. Show that the straight lines whose equations are $y = 2x$, $2y + x = 0$ and $2y = 2mx + 5$ form a right-angled triangle and find the coordinates of the vertices of this triangle.

Show also that if the triangle has an area of *5* sq. units then $m = \frac{3}{4}$

5. Find the equation of the straight line which intersects the coordinate axes at (a, 0) and (0, b).

The straight line $x + y = 3$ meets the axes of x and y in A and B respectively, while the straight line $3x + y = 3$ meets these axes in C and B respectively. The line joining *0,* the origin, to the mid-point of CB meets AB in P, while the line drawn from *0* perpendicular to CB meets AB in Q. Find the equations of the lines OP, OQ and prove that PQ is of length $\frac{3\sqrt{2}}{2}$ units.

6. The point *0* is the centre of the base of a right circular cone C_1 of given volume V cu. units and given base-radius r units. A second right circular cone C_2 is drawn inside C_1 with its vertex at *0* and with the edge of its base lying on the curved surface of C_1. Taking the base-radius of C_2 as tr units, where t can vary from 0 to 1, find an expression for the volume of C_2 in terms of t and V only. Hence show that the maximum volume of C_2 is $\frac{4}{27}V$ cu. units.

[The volume of a right circular cone of height h units and base radius r units is $\frac{1}{3}\pi r^2 h$ cu. units.]

[Turn over

7. (*a*) Find, from first principles, the differential coefficient with respect to *x* of $x^2 - 3x + 2$.

(b) The distance *s* moved in a straight line by a particle in time *t* is given by $s = at^2 + bt + c$, where a, *b* and *c* are constants. If *V* is the velocity of the particle at time *t*, show that
$V^2 - b^2 = 4a(s - c)$

8. Sketch the curve $y = \dfrac{x^2}{2a}$, where *a* is a positive number.

Find the area bounded by this curve, the line $x = 4a$, and the coordinate axes.

Show also that the volume of the solid of revolution generated when this area is rotated about the *y*-axis is $80\pi a^3$ cu. units.