UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

General Certificate of Education Advanced Level

MATHEMATICS **9200/1**

PAPER 1

Wednesday **5 JUNE 1996** Morning 3 hours

Additional materials:
- Answer paper
- List of Formulae
- Graph paper

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

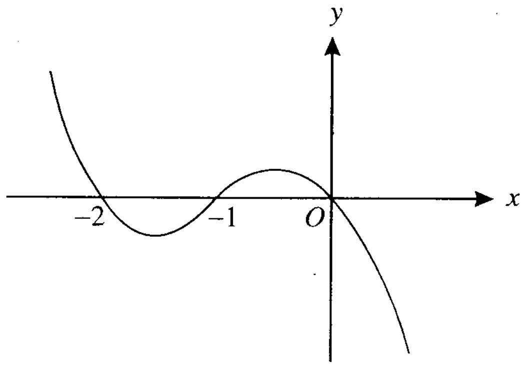
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

1 It is given that $(x + 2)$ is a factor of $x^4 - 4x^2 + 2x + a$. Find the value of the constant a. [2]

2



The graph of $y = f(x)$ is shown in the diagram. On separate diagrams sketch the graphs of

(i) $y = f(x + 1)$,

(ii) $y = |f(x + 1)|$,

showing the coordinates of the points where the graphs meet the x-axis. [3]

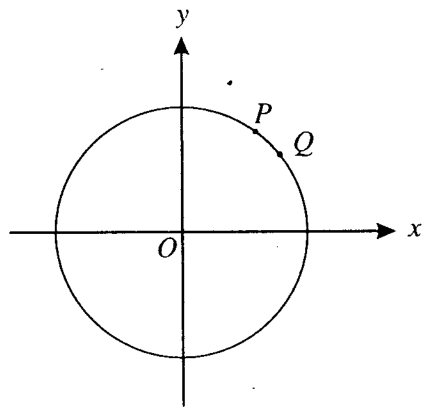
3 Expand $\frac{1}{(1-x)^2}$, where $|x| < 1$, in ascending powers of x, up to and including the term in x^3. You should simplify the coefficients. [2]

By putting $x = 10^{-4}$ in your expansion, find $\frac{1}{(0.9999)^2}$ correct to 12 decimal places. [1]

4 A cup has the shape made by rotating the graph of $y = 3x^2$, for $0 \leqslant x \leqslant 1$, through four right angles about the y-axis. Find the volume of the cup, giving your answer in terms of π. [4]

5 Express $\sin 4\theta$ in terms of $\sin 2\theta$ and $\cos 2\theta$, and hence express $\frac{\sin 4\theta}{\sin \theta}$ in terms of $\cos \theta$ only. [4]

6



The diagram shows the circle with equation $x^2 + y^2 = 25$. Find the length of the minor arc between the points $P(3, 4)$ and $Q(4, 3)$, giving your answer correct to 3 significant figures. [4]

7 Show that the equation $x = \dfrac{1}{2 + \sqrt{x}}$ has a root α between 0 and 1. [2]

By using an iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$, find α correct to two decimal places. You should show clearly your sequence of approximations. [3]

8 Find the x-coordinates of the stationary points of $y = x^3 \mathrm{e}^{-kx}$, where k is a positive constant. [5]

9 By means of the substitution $x = u^3$, show that

$$\int \frac{x^{-\frac{1}{3}}}{1 + x}\,\mathrm{d}x = \int \frac{3u}{1 + u^3}\,\mathrm{d}u.$$ [2]

Use the trapezium rule, with ordinates at $u = 0, \frac{1}{4}, \frac{1}{2}$, to find an approximate value for

$$\int_0^{\frac{1}{2}} \frac{3u}{1 + u^3}\,\mathrm{d}u,$$

giving your answer correct to two decimal places. [3]

10 A sequence is defined by $u_{n+1} = \dfrac{1}{1 - u_n}$, with $u_1 = a$, where $a \neq 1$. Find u_2, u_3 and u_4 in terms of a, simplifying your answers. [5]

Hence determine the behaviour of the sequence as n increases. [1]

11 **(a)** Given that the first and second terms of an arithmetic progression are 12 and 6 respectively, find the sum of the first hundred terms. [3]

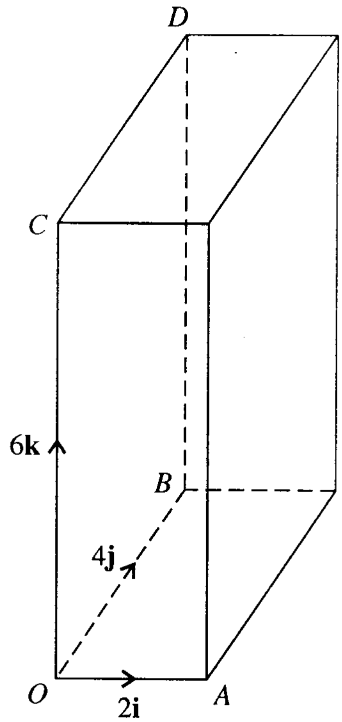
12 Express $\frac{3}{(2x+1)(x-1)}$ in partial fractions. [3]

Hence find the exact value of

$$\int_2^3 \frac{3}{(2x+1)(x-1)}\,dx,$$

giving your answer as a single logarithm. [5]

13



The three edges of a rectangular box that have a common vertex O are OA, OB, OC. The position vectors of A, B, C relative to O are $2\mathbf{i}$, $4\mathbf{j}$, $6\mathbf{k}$ respectively. The vertex opposite to A is D (see diagram).

(i) Find the position vector of the mid-point M of BD. [2]

(ii) Find a unit vector in the direction of the vector $\overrightarrow{OM}$. [2]

(iii) The point P inside the box has position vector $\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$. Find the angle CPA. [5]

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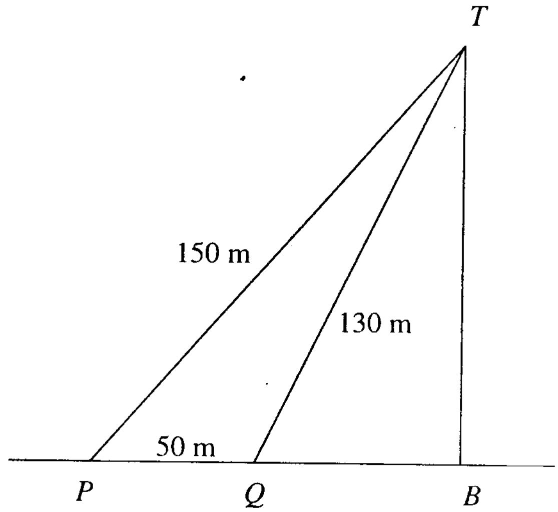

Fig. 1

A vertical radio mast BT is supported by straight wires that run from the top T of the mast to the ground. The wires TP and TQ have lengths 150 m and 130 m respectively and $PQ = 50$ m. In a simple model it is assumed that PQB is a horizontal straight line (see Fig. 1). By first finding the angle TQP, or otherwise, find the height BT of the mast, giving 3 significant figures in your answer. [5]

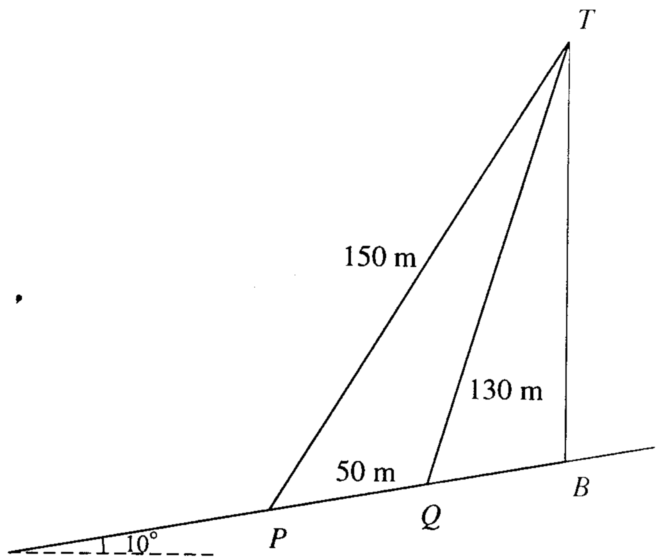

Fig. 2

Upon careful examination it is found that although PQB is a straight line it is not horizontal but is inclined at 10° to the horizontal, as shown in Fig. 2. Using this improved model, find the height BT of the mast, again giving 3 significant figures in your answer. [4]

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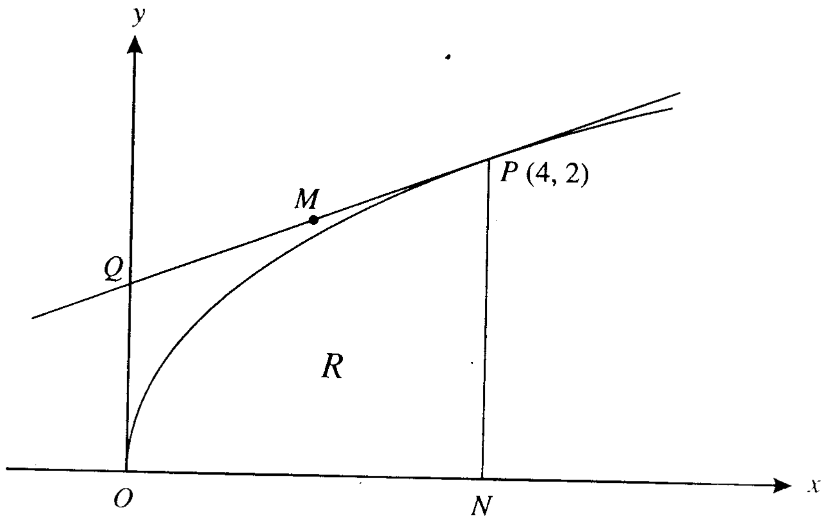

The diagram shows the graph of $y = x^{\frac{1}{2}}$. The point P on the graph has coordinates (4, 2). Find the equation of the tangent at P, giving your answer in the form $ax + by + c = 0$, where a, b, c are integers. [4]

The tangent at P meets the y-axis at Q. Find the coordinates of the mid-point M of PQ. [3]

Using integration, find the area of the region R bounded by the curve, the x-axis and PN, where PN is the perpendicular from P to the x-axis. [3]

16 (i) Carry out the process of completing the square for the quadratic polynomial $4x^2 - (5\sqrt{3})x$. Give your answer in either of the forms $A(x + B)^2 + C$ or $(Dx + E)^2 + F$. [3]

(ii) Solve the equation $4x^2 - (5\sqrt{3})x + 3 = 0$, giving your answers in exact form. [3]

(iii) Solve the equation $4y^4 - (5\sqrt{3})y^2 + 3 = 0$, giving your answers in exact form. [3]

(iv) Find the number of real roots of the equation $4x^2 - (5\sqrt{3})x + 5 = 0$. [2]

17 The rate of destruction of a drug by the kidneys is proportional to the amount of the drug present in the body. The constant of proportionality is denoted by k. At time t the quantity of drug in the body is x. Write down a differential equation relating x and t, and show that the general solution is $x = Ae^{-kt}$, where A is an arbitrary constant. [4]

Before $t = 0$ there is no drug in the body, but at $t = 0$ a quantity Q of the drug is administered. When $t = 1$ the amount of drug in the body is $Q\alpha$, where α is a constant such that $0 < \alpha < 1$. Show that $x = Q\alpha^t$. [3]

Sketch the graph of x against t for $0 < t < 1$. [2]

When $t = 1$ and again when $t = 2$ another dose Q is administered. Show that the amount of drug in the body immediately after $t = 2$ is $Q(1 + \alpha + \alpha^2)$. [2]

18 In a simple model of the tides in a harbour, the depth, y m, is given by

$$y = 20 + 5\cos t,$$

where t is the time measured in suitable units. For $0 \leqslant t \leqslant 4\pi$, write down the maximum and minimum values of y and the values of t when they occur. [4]

In a more refined model, y is given by

$$y = 20 + \{5 + \cos(\tfrac{1}{2}t)\}\cos t.$$

Using this model, show that the values of t when y is stationary satisfy the equation

$$\sin(\tfrac{1}{2}t)\{6\cos^2(\tfrac{1}{2}t) + 20\cos(\tfrac{1}{2}t) - 1\} = 0.$$ [5]

Hence find, correct to two decimal places, the stationary values of y for $0 \leqslant t \leqslant 4\pi$. [6]