UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

General Certificate of Education Advanced Level

MATHEMATICS 9200/2

PAPER 2

Tuesday **11 JUNE 1996** Morning 3 hours

Additional materials:
- Answer paper
- List of Formulae
- Graph paper

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ \mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

Section (a): Pure Mathematics

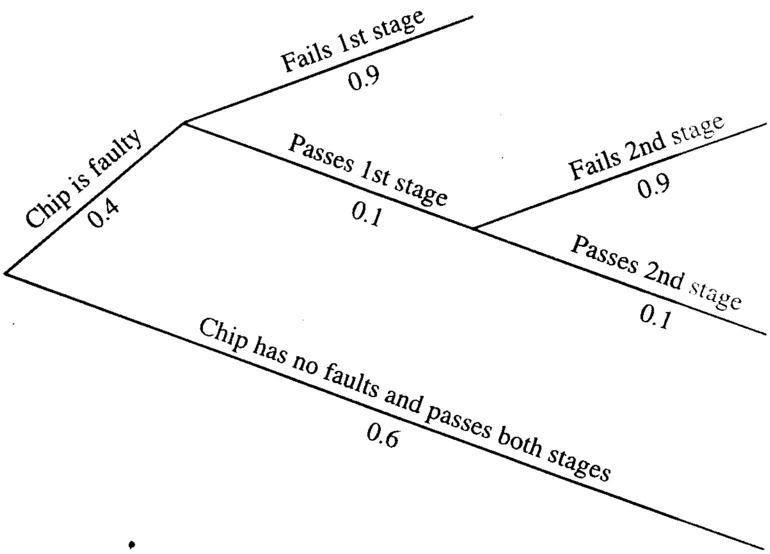
1 The parametric equations of a curve are

$$x = \ln t, \quad y = t + t^2,$$

where $t > 0$. Express $\dfrac{dy}{dx}$ in terms of t, simplifying your answer. [3]

2 Obtain the first three terms in the Maclaurin series for $\ln(3 + x)$. [5]

3 Computer chips are difficult to manufacture, and need to be checked for faults before they can be used. A test procedure has two stages, and in each stage the probability of a faulty chip failing is 0.9. Any chip that fails the first stage is immediately rejected and does not go through the second stage of the test. Chips with no faults always pass both stages of the test. The probability of any randomly chosen chip being faulty is 0.4. This information is illustrated in the following tree diagram.



(i) Find the probability that a randomly chosen chip is faulty and passes both stages of the test. [2]

(ii) Find the probability that a randomly chosen chip passes both stages of the test. [2]

(iii) Find the conditional probability that a randomly chosen chip is faulty, given that it passes both stages of the test. [2]

4 Two thousand amateur runners took part in a mini-marathon 'fun run' to raise money for charity. The first runner to finish took 54 minutes, and the table below shows the cumulative numbers of runners who had finished by various times.

Time after start (hours)	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4
No. finished by this time	23	457	1212	1684	1896	2000

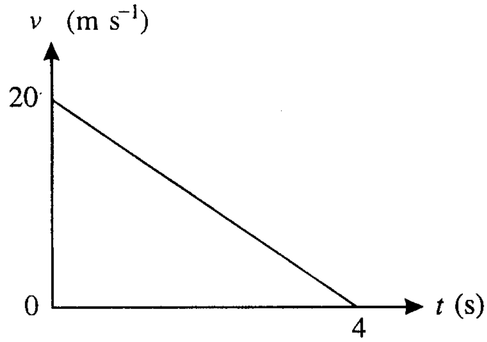
(i) Illustrate the data by drawing a cumulative frequency graph on graph paper. Use your graph to estimate the median time taken by the runners to finish and the interquartile range of the times. [4]

(ii) Calculate an estimate of the mean time taken by the runners to finish. [4]

(iii) By how much should your answer for the mean be modified, given the additional information that the last runner to finish took exactly $3\frac{1}{2}$ hours? [2]

Section (b): Mechanics

5



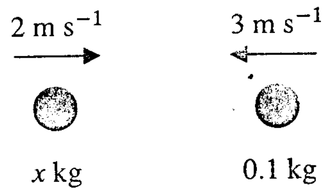
The diagram shows the (t, v) graph for the motion of a car of mass 600 kg which slows down uniformly from a speed of $20\,\mathrm{m\,s^{-1}}$ to rest in 4 s. The car is moving on a straight level road.

(i) Calculate the magnitude of the braking force that is applied to the car. [3]

(ii) Sketch a (t, v) graph for the motion of the car when the braking force applied is initially less than the value calculated in **(i)** but increases in magnitude as the car slows down. Assume that the initial speed of the car and the time for the car to stop remain as before. [2]

6 The force $\mathbf{R}$ is given by the vector $6\mathbf{i} + 2\mathbf{j}$, where the units of force are newtons. $\mathbf{R}$ is the resultant of a force $\mathbf{P}$ parallel to $\mathbf{i}$ and a force $\mathbf{Q}$ parallel to $\mathbf{i} + \mathbf{j}$. Find the magnitudes of $\mathbf{P}$ and $\mathbf{Q}$. [5]

7



Two particles, of masses x kg and 0.1 kg, are moving towards each other in the same straight line and collide directly. Immediately before the impact, the speeds of the particles are 2 m s^{-1} and 3 m s^{-1} respectively (see diagram).

(i) Given that both particles are brought to rest by the impact, find x. [2]

(ii) Given instead that the particles move with equal speeds of 1 m s^{-1} after the impact, find the three possible values of x. [5]

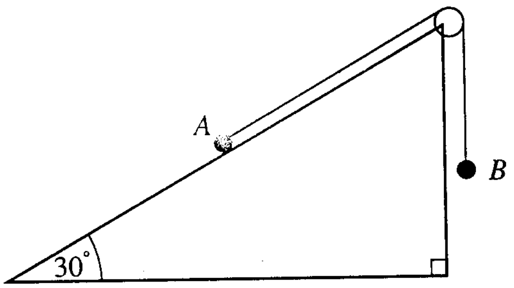
8 In a laboratory experiment the motion of a steel ball-bearing falling vertically in a tank containing a liquid is observed.

(i) State why the acceleration of the ball-bearing is less than g. [1]

(ii) The ball-bearing is released from rest in the liquid, and after 0.60 s it has fallen a distance of 1.53 m. Assuming that the acceleration of the ball-bearing has a constant value a m s^{-2}, find a and find also the speed of the ball-bearing after 0.60 s. [4]

(iii) At a time of 0.30 s after release the ball-bearing had fallen a distance of 0.41 m. Show how this observation contradicts the assumption of constant acceleration made in (ii). [2]

9



Particles A and B, each of mass m kg, are connected by a light inextensible string. Particle A rests on an inclined plane, particle B hangs freely, and the string passes over a smooth pulley at the top of the plane (see diagram).

(i) Given that the plane is smooth, show that the acceleration of each particle has magnitude $\frac{1}{4}g$ and express the tension in the string in terms of m and g. [5]

(ii) Given instead that the plane is rough and that the system is in limiting equilibrium, find the coefficient of friction between A and the plane. [4]

10 In the game of cricket, a player sometimes has to throw the ball from the edge of the field to another player in the middle of the field as quickly and accurately as possible. The motion of the ball is modelled as that of a particle moving under gravity with constant acceleration, and the two players are 60 m apart.

(i) A player throws the ball with a speed of $20\,\mathrm{m\,s^{-1}}$ at an angle of 45° above the horizontal from a height of 2 m above ground level. Show that the ball bounces before it reaches the other player 60 m away. [5]

(ii) A player who can throw the ball with a speed of $25\,\mathrm{m\,s^{-1}}$ throws it with this speed, from a height of 2 m above ground level, at an angle α above the horizontal. The ball travels without bouncing to the other player 60 m away who catches it at a height of 1 m above ground level. Using the Cartesian equation of the trajectory, or otherwise, find the two possible values of α, and find the time taken for the throw when the more suitable of the two values for α is used. [8]

(iii) State what force is ignored in the mathematical model used in **(i)** and **(ii)**. [1]

(iv) In order to find, for the throw in **(i)**, the total time taken before the ball reaches the player in the middle of the field, it would be necessary to have some further information (apart from including the effect of the force in **(iii)**). Describe briefly what further information is required. [1]

Section (c): Statistics

11 The random variable X has a normal distribution with mean 6 and standard deviation 4.5.

(i) Calculate $\mathrm{P}(X > 0)$. [3]

(ii) The distribution of X is proposed as a model for the length of time, measured in hours, for which a certain drug remains effective after being given to a randomly chosen patient. Comment on the suitability of this model with reference to your answer in **(i)**. [2]

12 A fair coin is tossed 80 times. Use an appropriate normal distribution to estimate the probability that the number of heads obtained lies between 35 and 45 inclusive. [6]

13 An unbiased cubical die has three faces numbered '1', two faces numbered '2' and one face numbered '3'. The random variable X is the number showing on the top face of the die when it is thrown. Show that $\mathrm{E}(X) = \frac{5}{3}$, and find $\mathrm{Var}(X)$. [6]

14 Each time a player plays a certain card game called 'patience', there is a probability of 0.15 that the game will work out successfully and there is a probability of 0.85 that the game will not work out successfully. The probability of success in any game is independent of the outcome of any other game.

(i) A player plays six games of patience.

(a) Calculate the probability that exactly one of the games works out successfully. [3]

(b) Write down the expected number of games that work out successfully. [1]

(ii) A player plays games of patience until a game works out successfully.

(a) Calculate the probability that at most six games are played. [3]

(b) Write down the expected number of games that are played. [1]

15 The 'reading age' of children about to start secondary school is a measure of how good they are at reading and understanding printed text. A child's reading age, measured in years, is denoted by the random variable X. The distribution of X is assumed to be $N(\mu, \sigma^2)$.

(i) The reading ages of a random sample of 20 children were measured, and the data obtained is summarised by $\Sigma x = 232.6$, $\Sigma x^2 = 2756.22$. Calculate unbiased estimates of μ and σ^2, giving your answers correct to 1 decimal place. [3]

(ii) The mean reading age of a random sample of 25 children was 11.4 years. Given that $\sigma^2 = 2.25$, calculate a 95% confidence interval for μ, stating the end-points of your interval correct to 1 decimal place. [3]

(iii) A random sample of 100 children is taken. Given that $\mu = 11.5$ and $\sigma^2 = 2.25$, calculate the probability that the mean reading age of the sample is greater than 11.65 years. [4]

16 The table below shows, for each of the years 1985 to 1991, the average unemployment rate x (expressed as a percentage) and the number of crimes committed y (measured in millions). The data refers to Scotland.

Year	1985	1986	1987	1988	1989	1990	1991
Unemployment rate, x	12.9	13.3	13.0	11.3	9.3	8.1	8.7
No. of crimes, y	0.80	0.82	0.86	0.86	0.90	0.96	1.02

[$\Sigma x = 76.6$, $\Sigma x^2 = 867.78$, $\Sigma y = 6.22$, $\Sigma y^2 = 5.5636$, $\Sigma xy = 67.144$.]

Plot a scatter diagram on graph paper, showing unemployment rate on the horizontal axis and number of crimes on the vertical axis. [3]

Calculate the product moment correlation coefficient between x and y, and interpret the result of the calculation in terms of your scatter diagram. [3]

The number of years after 1985 is denoted by t. Calculate the equation of the regression line of y on t, giving your answer in the form $y = a + bt$, where a and b are constants to be determined. [4]

Use your equation to estimate the number of crimes committed in 1992, and discuss briefly the likely reliability of this estimate. [3]