UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

Joint Examination for the Higher School Certificate and General Certificate of Education Advanced Level

MATHEMATICS 9200/1

PAPER 1

Tuesday **10 NOVEMBER 1998** 3 hours

Additional materials:
- Answer paper
- List of Formulae
- Graph paper

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

1 The straight line L passes through the point $(3, -2)$ and is perpendicular to the line $x + 2y + 1 = 0$. Find the equation of L, giving your answer in the form $y = mx + c$. [3]

2 Given that $|x + a| = |x|$, where a is a non-zero constant, express x in terms of a. [3]

3 The power, P kilowatts, needed from a car's engine to drive the car at its maximum speed of $v\,\text{km}\,\text{h}^{-1}$ on a level road is directly proportional to v^3. Calculate the percentage increase in power needed from the engine if the car's maximum speed is to be raised from $100\,\text{km}\,\text{h}^{-1}$ to $110\,\text{km}\,\text{h}^{-1}$. [3]

4 Find $\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x$, giving your answer in terms of e. [4]

5 The cubic polynomial $2x^3 + ax^2 - x - 2$ has a factor $(x - 2)$. Find the value of the constant a. [2]

Show that, for this value of a, the equation

$$2x^3 + ax^2 - x - 2 = 0$$

has only one real root. [3]

6 Sketch the graph of $y = |\tan x|$, for $0 \leqslant x \leqslant 2\pi$, showing clearly the positions of the asymptotes. [2]

Solve the inequality $|\tan x| < 1$ for $0 \leqslant x \leqslant 2\pi$. [3]

7 Expand $(1 - 2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2. [3]

State the set of values of x for which the expansion is valid. [1]

Calculate the relative error in using these terms of the expansion as an approximation for $(1 - 2x)^{-\frac{1}{2}}$ when $x = 0.1$. [2]

8 Express 4^x in terms of y, where $y = 2^x$. [1]

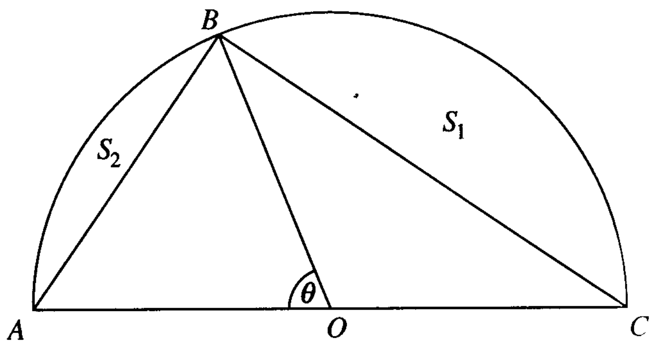
Solve the equation

$$6(4^x) - 5(2^x) + 1 = 0,$$

expressing your answers for x in terms of logarithms where appropriate. [5]

9 The straight line $y = 20 - 3x$ meets the circle $x^2 + y^2 - 2x - 14y = 0$ at the points A and B. Calculate the exact length of the chord AB. [6]

10



The diagram shows a semicircle ABC on AC as diameter. The mid-point of AC is O, and angle $AOB = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$. The area of the segment S_1 bounded by the chord BC is twice the area of the segment S_2 bounded by the chord AB. Show that

$$3\theta = \pi + \sin\theta.$$ [3]

Use the iterative formula

$$\theta_{n+1} = \tfrac{1}{3}(\pi + \sin\theta_n),$$

together with a suitable starting value, to find θ correct to 3 significant figures. You should show the value of each approximation that you calculate. [3]

11 The parametric equations of a curve are

$$x = t - \frac{1}{t}, \quad y = t - \frac{2}{t},$$

where $t > 0$.

(i) Find $\frac{dy}{dx}$ in terms of t, and hence find the value of $\frac{dy}{dx}$ when $x = 0$. [5]

(ii) Write down the first two terms of the Maclaurin series for y in terms of x. [1]

12 Points A, B and C have coordinates (4, 0, 4), (0, 6, 6) and (0, 0, c) respectively. The point O is the origin, and the mid-point of AB is M.

(i) Find the vectors $\overrightarrow{OM}$ and $\overrightarrow{CM}$. [2]

(ii) Given that $c = 5$, calculate angle OMC. [3]

(iii) Find the value of c for which angle OMC is a right angle. [2]

13 The function f is defined by

$$\mathrm{f} : x \mapsto \ln(x+1), \quad x > -1.$$

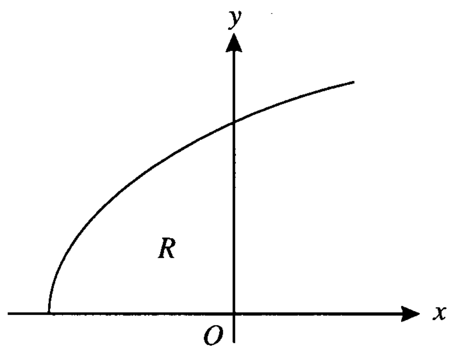
Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of the inverse function f^{-1}. [4]

The function g is defined by

$$\mathrm{g} : x \mapsto x - 1, \quad x \in \mathbb{R}.$$

Describe the geometrical relationship between the graphs of $y = \mathrm{fg}(x)$ and $y = \mathrm{gf}(x)$. [4]

14



The diagram shows the region R bounded by the axes and part of the curve $y = \sqrt{(x+1)}$. Find the exact value of

(i) the area of R, [4]

(ii) the volume of the solid formed when R is rotated completely about the y-axis, giving your answer as a multiple of π. [4]

15 **(i)** A sequence of positive integers $u_1, u_2, u_3, \ldots$ is given by

$$u_1 = 2 \quad \text{and} \quad u_{n+1} = 2u_n \text{ for } n \geqslant 1.$$

(a) Write down the first four terms of this sequence. [1]

(b) State what type of sequence this is, and express u_n in terms of n. [2]

(ii) A sequence of positive integers $v_1, v_2, v_3, \ldots$ is given by

$$v_1 = 3 \quad \text{and} \quad v_{n+1} = 2v_n - 1 \text{ for } n \geqslant 1.$$

(a) Show that the relation between v_{n+1} and v_n may be written in the form

$$v_{n+1} - 1 = 2(v_n - 1).$$ [1]

(b) Hence, by using the results in part **(i)**, show that $v_n = 2^n + 1$ for $n \geqslant 1$. [2]

(iii) Express $\sum^{N} v_n$ in terms of N. [4]

16 (a) Find the general solution of the differential equation

$$\frac{dx}{dt} = kx,$$

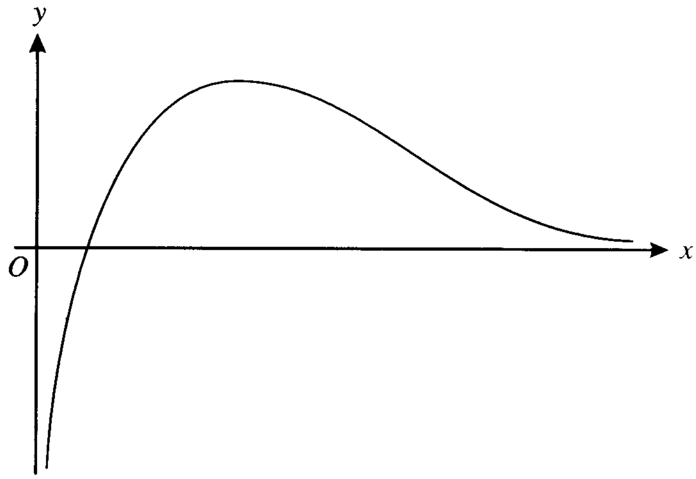
where k is a positive constant, expressing x in terms of k and t in your answer. [4]

(b) Solve the differential equation

$$\frac{dx}{dt} = kx(a - x) \qquad (0 < x < a),$$

where k and a are positive constants, given that $x = \frac{1}{2}a$ when $t = 0$. Express x in terms of k, a and t in your answer. [6]

17



The diagram shows a sketch of the curve $y = \frac{\ln x}{x}$. Use differentiation to show that the maximum value of y occurs when $x = e$. [3]

State the set of values of the constant k for which the equation

$$\frac{\ln x}{x} = k$$

has two distinct real roots for x. [2]

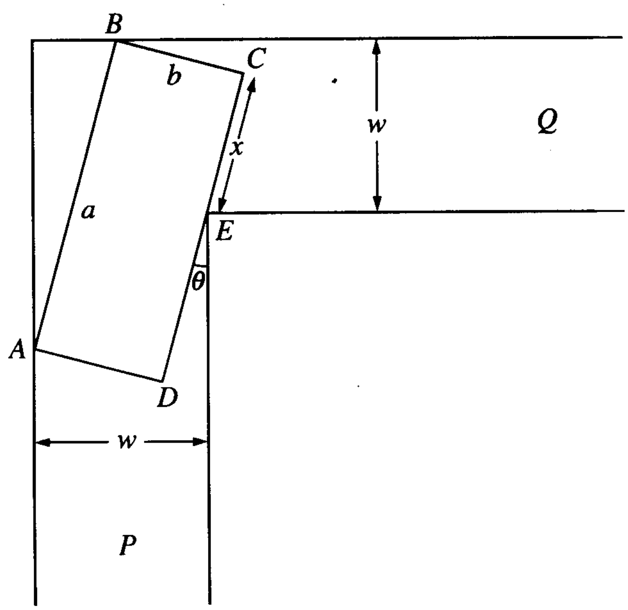
These roots are denoted by a and b, where $a < b$.

(i) Explain why $1 < a < e$, and state an inequality satisfied by b. [3]

(ii) Show that $a^b = b^a$. [2]

(iii) Given that a and b are positive integers, deduce from parts (i) and (ii) the values of a and b. [2]

18



Two straight corridors, P and Q, each of width w, meet at right angles. $ABCD$ is a rectangular crate of length a and breadth b. In the position shown in the diagram, the angle between DC and the wall of corridor P is θ. The crate touches the outer walls at A and B, and touches the inside corner at E, where $CE = x$.

(i) Show that $x\cos\theta + b\sin\theta = w$, and find an equation relating a, b, x, w and θ. [3]

(ii) By eliminating x from the equations in part **(i)**, or otherwise, show that

$$\tfrac{1}{2}a\sin 2\theta + b = w(\sin\theta + \cos\theta).$$ [3]

(iii) Let $\theta = 45° - \phi$. Show that the equation in part **(ii)** may be expressed as

$$a\cos^2\phi - (w\sqrt{2})\cos\phi + (b - \tfrac{1}{2}a) = 0.$$ [3]

(iv) Find θ in the case when $a = 4$, $b = 1$ and $w = 2$. [3]