UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

Joint Examination for the Higher School Certificate and General Certificate of Education Advanced Level

MATHEMATICS 9200/2

PAPER 2

Thursday **12 NOVEMBER 1998** 3 hours

Additional materials:
Answer paper
List of Formulae
Graph paper

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ \mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

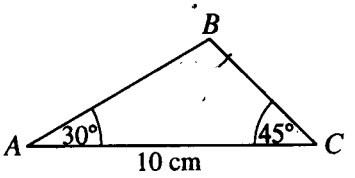
Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

Section (a): Pure Mathematics

1

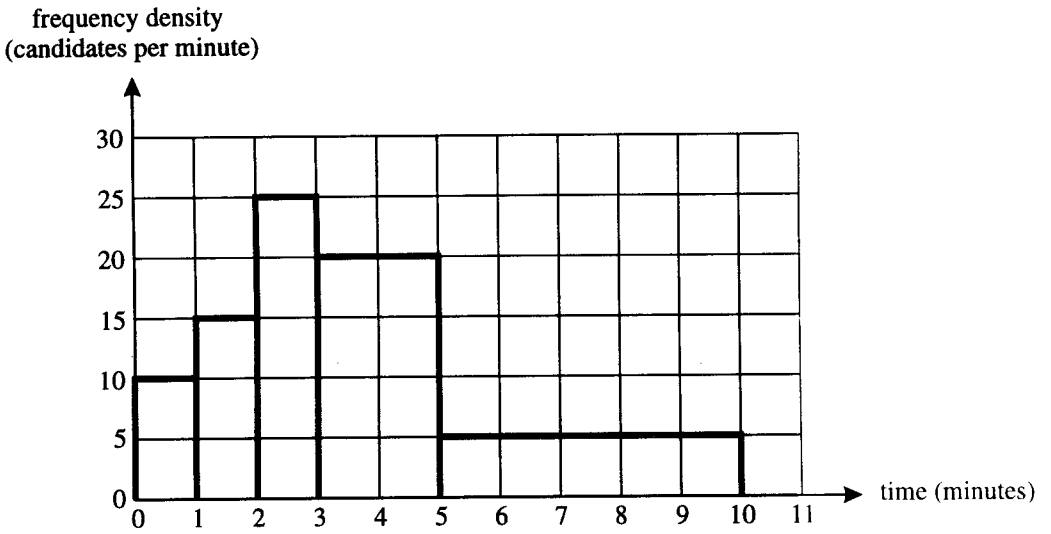


The diagram shows triangle ABC, in which $AC = 10$ cm, angle $BAC = 30°$ and angle $BCA = 45°$

(i) Expand $\sin(45° + 30°)$ and hence express $\sin 75°$ in surd form. [3]

(ii) Show that the exact length of AB is $\dfrac{20}{1+\sqrt{3}}$ cm. [4]

2



An intelligence test was taken by 115 candidates. For each candidate the time taken to complete the test was recorded, and the times were summarised in a histogram (see diagram). Write down the frequency for each of the class intervals $0 - 1$, $1 - 2$, $2 - 3$, $3 - 5$ and $5 - 10$ minutes. [1]

Calculate estimates of the mean and standard deviation of the times taken to complete the test. [6]

3 A die is known to be biased in such a way that, when it is thrown, the probability of a 6 showing is $\frac{1}{4}$. This biased die and an ordinary fair die are thrown. Find the probability that

(i) the fair die shows a 6 and the biased die does not show a 6, [3]

(ii) at least one of the two dice shows a 6, [3]

(iii) exactly one of the two dice shows a 6, given that at least one of them shows a 6. [4]

Section (b): Mechanics

4

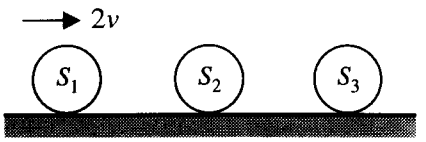


Fig. 1

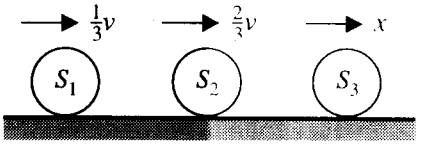
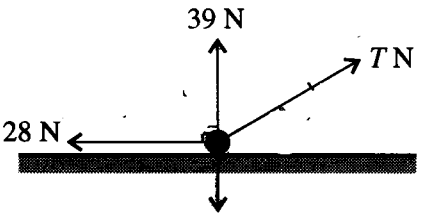


Fig. 2

Three identical smooth uniform spheres S_1, S_2 and S_3, each of mass m, lie at rest in a straight line on a horizontal plane. S_1 is projected with speed $2v$ towards S_2 (see Fig. 1). After the collisions between S_1 and S_2, and between S_2 and S_3, the speeds of S_1, S_2 and S_3 are $\frac{1}{3}v$, $\frac{2}{3}v$ and x respectively (see Fig. 2). Find, in terms of v, the speed of S_2 in the period between the two collisions, and express x in terms of v. [4]

5



A particle of weight 45 N rests on a rough horizontal surface. A cord is attached to the particle and the particle is in limiting equilibrium when the tension in the cord is T newtons. The diagram illustrates the forces acting on the particle. Calculate the coefficient of friction and the magnitude of the contact force between the particle and the surface. [3]

Calculate T and the angle between the cord and the horizontal. [4]

6 Sketch (t, v) graphs for each of the following situations.

(i) A car starts from rest and moves with constant acceleration until the driver changes gear. For a short time, while the driver changes gear, the speed is constant. After the driver changes gear, the car continues with a constant acceleration which is less than the initial acceleration. [2]

(ii) A swimmer completes one length of a pool, turns, and completes one length in the opposite direction, swimming at constant speed throughout, except for a brief reduction in speed at the turn. [3]

Sketch also a (t, x) graph for situation **(ii)**. [2]

7

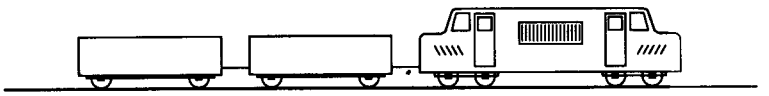


Fig. 1

Fig. 1 shows a railway engine of mass 50 tonnes pulling two loaded trucks horizontally along a straight track. The trucks are coupled together behind the engine and have masses 8 tonnes and 4 tonnes respectively, starting with the truck nearer to the engine. The acceleration of the train is $0.5\,\mathrm{m\,s^{-2}}$. Assuming there are no resistances to motion, find the driving force of the engine. [2]

Find also the tensions in the two couplings. [3]

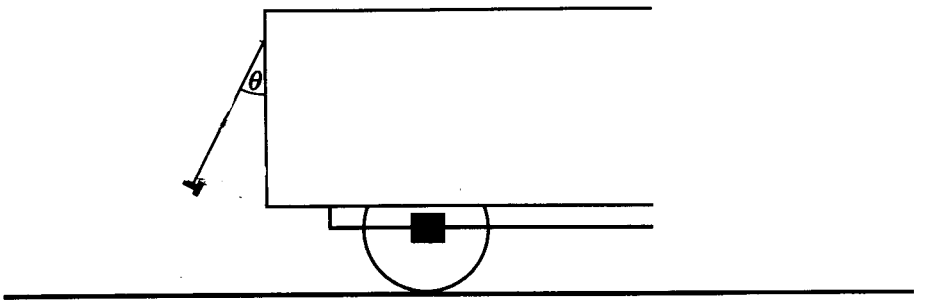


Fig. 2

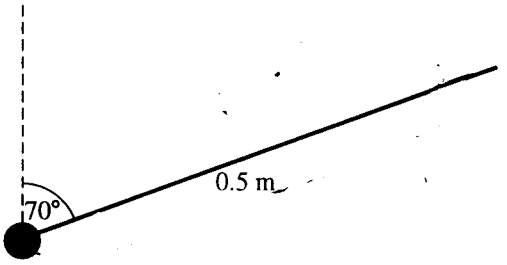
A small metal bolt is attached to one end of a short light metal chain. The other end of the chain is attached at the back of the rear truck, and the bolt hangs freely, with the chain at a constant angle of θ to the vertical, as shown in Fig. 2. Neglecting air resistance, calculate θ. [4]

8 Two friends are standing at different levels in a garden, and one throws a ball to the other. Their horizontal distance apart is 8 m and the angle of projection is 60° above the horizontal. The effect of air resistance is neglected.

(i) It is given that the ball is caught at a height of 1.5 m above the level of the point from which it is thrown. Find the speed of projection. [5]

(ii) It is given instead that the direction of motion of the ball, when it is caught, is 20° below the horizontal. Find the speed of projection. [5]

9



A straight rough wire is fixed at an angle of 70° to the vertical and has a rubber stopper attached at its lower end. A small bead of mass 0.01 kg is threaded onto the wire. The coefficient of friction between the bead and the wire is μ. The bead is released from rest at a distance of 0.5 m from the stopper (see diagram).

(i) For the case $\mu = 0.3$, find the acceleration of the bead and show that its speed just before it strikes the stopper is $0.77\ \text{m s}^{-1}$, correct to two significant figures. [5]

(ii) For a different value of μ it is found that, after rebounding from the stopper without loss of speed, the bead first comes to instantaneous rest at a distance of 0.25 m from the stopper. Find the value of μ. [6]

Section (c): Statistics

10 The makers of a lime-flavoured drink wish to find out what people think of their drink, and decide to interview a sample of shoppers. Comment briefly on the suitability of each of the following samples.

(i) A sample of shoppers who have just bought the drink. [1]

(ii) A sample of shoppers consisting of one person aged 20, one person aged 21, one person aged 22, and so on, up to one person aged 80. [1]

(iii) A sample consisting of every 50th shopper. [1]

11 It is given that $X \sim \text{B}(16, 0.5)$.

(i) Calculate $\text{P}(X = 4)$ directly, using the binomial distribution. [2]

(ii) Calculate $\text{P}(X = 4)$ using a normal approximation. [4]

12 A hotel manager is reviewing the number, B, of bars of soap used daily in the hotel. From an analysis of records taken over a long period of time she finds the following probabilities:

$$\text{P}(B \geqslant 120) = 0.71, \qquad \text{P}(B \geqslant 200) = 0.14.$$

The manager decides to treat B as a continuous random variable and to use a normal model for the distribution of B. Find, correct to the nearest whole number, the mean and standard deviation of B. [7]

13 A catering company asked 50 randomly selected college students to state the amount of money, $\$x$, which they spent daily on lunch, and the results were summarised by $\Sigma x = 56.50$ and $\Sigma x^2 = 66.80$. Calculate unbiased estimates of the mean and the variance of the amount spent daily on lunch by students at the college, giving your answers correct to three significant figures. [3]

Hence find a symmetric 90% confidence interval for the mean amount spent daily on lunch, giving the end-points correct to the nearest \$0.01. [4]

Justify the use of the normal distribution in constructing the confidence interval. [1]

14 The following table gives x, the number of hours of sunshine, and y, the mid-day temperature in °C, at Springtown on the first 7 days in May.

Date	May 1st	May 2nd	May 3rd	May 4th	May 5th	May 6th	May 7th
Hours of sunshine x	10	11	2	7	5	6	12
Mid-day temperature y °C	17	21	12	13	18	16	15

$[\Sigma x = 53,\ \Sigma y = 112,\ \Sigma x^2 = 479,\ \Sigma y^2 = 1848,\ \Sigma xy = 882.]$

Plot the data on a scatter diagram. [2]

Calculate the product moment correlation coefficient. [2]

The regression line of x on y has equation $x = 0.607y - 2.14$, and the regression line of y on x has equation $y = 0.438x + 12.7$, where the coefficients are correct to 3 significant figures. Using the equation of the appropriate regression line, estimate the number of hours of sunshine expected on a day in May when the mid-day temperature is 18 °C. [2]

Give a reason why this estimate differs from the actual number of hours of sunshine on May 5th. [1]

Explain the concept of least squares by reference to your scatter diagram and the regression line of y on x. [2]

15 Show that, when two fair dice are thrown, the probability of obtaining a 'double' is $\frac{1}{6}$, where a 'double' is defined as the same score on both dice. [1]

Four players play a board game which requires them to take it in turns to throw two fair dice. Each player throws the two dice once in each round. When a double is thrown the player moves forward six squares. Otherwise the player moves forward one square. Find

(i) the probability that the first double occurs on the third throw of the game, [2]

(ii) the probability that exactly one of the four players obtains a double in the first round, [2]

(iii) the probability that a double occurs exactly once in 4 of the first 5 rounds, [2]

(iv) the variance of the number of squares moved by a single player in one throw. [5]

The first round in which no player obtains a double is the Xth round. Find the variance of X. [3]