University of Nottingham

FACULTY OF PURE SCIENCE

SCHOOL OF PHYSICS

PART II EXAMINATION, 1968

PHYSICS (iii)

WEDNESDAY *June 5th* 2.30 - 5.30

Answer FOUR *questions*

1. Prove from Maxwell's equations that electromagnetic waves in free space are transverse and describe in general terms how this theory accounts for the reflection and refraction of electromagnetic waves at a dielectric surface.

State the relationship between the energy flow in an electromagnetic wave and the electric and magnetic fields. An elliptically polarised beam with

$$\mathbf{E} = E_0 \, \mathbf{i} \sin\omega(t - \frac{z}{c}) + E_0 \, \mathbf{j} \, \sin\{\omega(t - \frac{z}{c}) + \frac{\pi}{4}\},$$

is incident normally from free space on a perfectly absorbing surface. What is the value of E_0 if the difference between the maximum and minimum energy being absorbed by the surface is 1.4 J m^{-2} sec^{-1}.

[$\mu_o = 4\pi \times 10^{-7}$ H m^{-1}, $\varepsilon_o = 8.8 \times 10^{-12}$ F m^{-1}.]

2. Describe without detailed mathematics how the transmission of electrornagnetic radiation in a continuous conductor differs from that in a continuous dielectric and outline some practical consequences of these differences.

Explain why at very low temperatures pure normal metals cannot be considered as continuous ohmic conductors when exposed to microwave or even radiofrequency radiation.

An oscillatory electromagnetic field is incident normally on the plane surface of a large block of a good conductor. Obtain expressions for the density of eddy currents in the block and the total current per unit width of the surface flowing in the block. The displacement current may be neglected and the dimensions of the block may be assumed much greater than the skin depth.

3. From the retarded magnetic vector potential solution of Maxwell's equations, derive an expression for the magnetic induction **B** at a distance r from a short length 1 of wire carrying a current $i = i_o \cos \omega t$.

By making use of this result and the reciprocity theorem determine **B** at large distances from a circular wire loop of diameter l. Hence deduce that the radiation resistance of the loop is $\pi^5 Z_0 l^4 / 6\lambda^4$, where Z_0 is the characteristic impedance of free space and λ is the wavelength.

4. Discuss the reasons for the use of transmission lines and waveguides at very high frequencies.

A parallel wire transmission line of length 100 m is terminated by a 300 Ω load. Show that the r.m.s. line current I at a distance z from the source is given by

$$\frac{I^2}{I_L^2} = \cos^2 \beta(l - z) + \left(\frac{Z_L}{Z_0}\right)^2 \sin^2 \beta(l - z),$$

where I_L is the load current, β is the phase constant, 1 is the length of the line, Z_L is the load impedance and Z_0 is the characteristic impedance of the line. If the line parameters are

$R = 0.025$ Ω rn^{-1}, $L = 2$ μH m^{-1}, $G = 0$, $C = 556$ pF m^{-1}

calculate the efficiency of transmission at 4 Mc/s.

[Turn over

5. An effectively infinite network is composed of identical T sections each having series reactances $\frac{1}{2}X_1$ and a parallel reactance X_2. Neglecting resistive losses, show that signals of all frequencies for which $X_1/4X_2$ lies between 0 and -1 will be passed freely by this network.

Each T section in such a network has series inductances each of value $3L/10$ and a shunt element consisting of an inductance of value $4L/15$ in series with a condenser of capacity $3C/5$. Show that this network is a low pass filter with a cut-off frequency given by $f_c = 1/\pi\sqrt{(LC)}$, and that the transmission in its stop band is zero at the frequency $5f_c/4$. Discuss the disadvantages which this filter may have.

6. State and define the electrostatic boundary conditions which apply at the surface of (*a*) a dielectric and (*b*) a conductor. Explain how they are used in the method of images for solving electrostatic problems.

Two large plane conducting sheets are earthed and joined together along one edge so that the planes are mutually perpendicular. A charge Q is at a distance a from one plane and b from the other with $a < b$. Find an expression for the density of charge on the conducting surface at the point nearest to the charge Q. Obtain also the energy necessary to move the charge Q to a great distance from the planes. Derive any formulae you need from first principles.

7. Show how a four dimensional electromagnetic vector potential may be set up from Maxwell's equations and how the components of the electromagnetic field tensor may be derived from it. You may assume the operator identity

$$\text{curl curl} = \text{grad div} - \nabla^2$$

Derive an expression for the ratio B'_x / E'_y as measured in a reference frame S' in terms of the electromagnetic field components measured in reference frame S if S' moves with velocity v along the x axis of S.

8. State Ampere's law for the magnetic field due to a current and use it to derive an expression for the field inside a long solid cylindrical conductor carrying a steady current i. Describe the forces experienced by the conductor due to the current which it carries.

A long solid cylinder of radius a is divided into two halves along a diameter, but the halves are not separated. The cylinder carries a current i uniformly distributed across the whole cross section. Show that each half experiences a force per unit length given by $\mu\mu_0 i^2 / 3\pi a^2$.

Comment on the current distribution in the conductor in the case of very high current density.