

University of Nottingham

FACULTY OF PURE SCIENCE

SCHOOL OF PHYSICS

PART II EXAMINATION, 1968

PHYSICS (iv)

THURSDAY *June 6th* 2.30 - 5.30

Answer FOUR questions

1. Derive the four Maxwell relations appropriate to an elastic rod of length L under a compressive force D .

Show that in principle such a system could be used to correct an arbitrary scale of temperature T^* to the Kelvin scale of temperature. Indicate what experimental difficulties would be met in practice.

The following properties of a solid rod were determined near to the ice point using an arbitrary temperature scale T^* :

- (a) total heat capacity = $10^8 \text{ erg deg}^{-1}$,
- (b) linear thermal expansion coefficient = 10^{-5} deg^{-1} ,
- (c) isentropic temperature change on compression = $-10^{-11} \text{ (deg dyne}^{-1}\text{)}$.

Calculate the Kelvin temperature corresponding to $T^* = 1^\circ$ above the ice point.

[Ice point = $273.16 \text{ }^\circ\text{K}$.]

2. Obtain the Clausius—Clapeyron relation

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

relating the molar latent heat of vaporisation L of a liquid, to the vapour pressure P at temperature T and the molar volumes V_2 and V_1 of the vapour and liquid respectively.

Show that over a range of temperature from T_1 to T_2 well below the critical temperature, the fractional change in the vapour pressure is given approximately by the expression

$$\frac{P_2}{P_1} = \exp \frac{L}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

where R is the gas constant. Assume that the latent heat remains constant over this range of temperature.

Why would this expression become a very poor approximation to the experimental data near to the critical point of the substance?

A saturated vapour is expanded isentropically. Show it becomes supersaturated or unsaturated according to whether $-T(d/dT)(L/T)$ is greater or less than the specific heat of the liquid under its own vapour pressure.

3. Distinguish between the terms adiabatic, isenthalpic and isentropic as applied to the expansion of gas. Show that the temperature of any gas may be expected to fall in an isentropic expansion.

The usual way of cooling liquid helium contained in a Dewar flask is to cause it to boil under reduced pressure by pumping away the vapour. The latent heat of evaporation of liquid helium 23 J gm^{-1} and the specific heat of the liquid is given as a function of temperature in the table below. What fraction of the liquid must be lost in reducing the temperature from the normal boiling point of $4 \text{ }^\circ\text{K}$ to $3 \text{ }^\circ\text{K}$?

$T(^\circ\text{K})$	4.5	4.0	3.5	3.0	2.5
$C (\text{Jgm}^{-1} \text{ deg}^{-1})$	4.9	3.8	3.0	2.5	2.2

4. Use the Maxwell-Boltzmann partition function to derive the classical expression $\bar{\epsilon} = \frac{1}{2}kT$ for the mean thermal energy associated with each degree of freedom of a system in equilibrium at a temperature T . State what physical assumptions are made concerning the system.

Explain why the equipartition of energy theorem accounts well at all temperatures for the specific heat of a monatomic gas but fails in the case of solids and polyatomic gases.

$$\left[\int_{-\infty}^{\infty} \exp(-ax^2) dx = \sqrt{\frac{\pi}{a}} \right]$$

5. What microscopic properties must be possessed by a set of particles if they are to be distributed amongst a set of states j of energy ϵ_j according to the expression

$$\bar{n}_j = \frac{1}{\exp\{(\epsilon_j - \mu)/kT\} - 1}$$

where k is the Boltzmann constant, T is the temperature and μ is the chemical potential per particle?

Sketch the form of this distribution function emphasizing how it differs from the Maxwell-Boltzmann distribution.

The allowed energy states (n_1, n_2, n_3) of a particle of mass m in a box of linear dimensions l_1, l_2 and l_3 may be shown to have energies

$$\epsilon_{n_1 n_2 n_3} = \frac{\pi^2 (h/2\pi)^2}{2m} \left(\frac{n_1^2}{l_1^2} + \frac{n_2^2}{l_2^2} + \frac{n_3^2}{l_3^2} \right).$$

Show that the distribution function given above approximates to the Maxwell-Boltzmann distribution in the case of helium gas at 20 °K but not in the case of liquid helium at 3 °K.

[Atomic density of He gas at 20 °K = 2×10^{20} atom cm^{-3} ,
atomic density of He liquid at 3 °K = 2×10^{22} atom cm^{-3} ,
mass of He atom = 6.6×10^{-24} gm, $k = 1.4 \times 10^{-16}$ erg deg^{-1} , $h/2\pi = 1.0 \times 10^{-27}$ erg sec.]

6. State the relationship between the partition function and the entropy for a system consisting of a set of independent, distinguishable particles. Use this expression to calculate the entropy and heat capacity of a solid consisting of N atoms, half of which have a natural frequency of vibration of ν and half have a natural frequency of 2ν .

Show that the entropy of such a solid in the high temperature limit may be written as

$$S = \frac{3Nk}{2} \left(2 + \ln \frac{T^2}{2\theta^2} \right),$$

where $\theta = h\nu/k$ and k is the Boltzmann constant.

Obtain an expression for the temperature dependence of the specific heat in the high temperature region and sketch it as a function of T .

Explain why such a model may be expected to fit the experimental data of a real solid better than the simple Einstein model but not so well as the Debye model.

7. Given operators

$$A = x + \frac{d}{dx}, \quad B = x - \frac{d}{dx}$$

show that

(a) $AB - BA = 2$, (b) $A(AB + BA) - (AB + BA)A = 4A$, (c) $B(AB + BA) - (AB + BA)B = -4B$.

Deduce that if $f(x)$ is an eigenstate of $(AB + BA)$ with eigenvalue λ , $Af(x)$ is another eigenstate and find its eigenvalue.

Verify that $K \exp(-\frac{1}{2}x^2)$, where K is a constant, is an eigenstate of $x^2 - \frac{d^2}{dx^2}$ and find another eigenstate.

8. Explain what is meant by a normalised wave function and indicate why normalisation is important. Illustrate your answer by discussing the wave mechanical treatment of a particle moving freely in an infinite volume.

Indicate qualitatively what you would expect the description to be if the particle is charged and there is a uniform magnetic field throughout the volume.

9. A particle of mass m moves under an inverse square law of attraction towards the origin and one of its un-normalised eigenfunctions is, in spherical polar co-ordinates and with suitable units,

$$r e^{-\frac{1}{2}r} \cos\theta .$$

Verify that this is an eigenstate of L_z , the operator corresponding to the component of angular momentum about the polar axis, and determine the expectation values of

- (a) the component of angular momentum about the line $\theta = \frac{1}{2}\pi$, $\phi = 0$,
- (b) the kinetic energy, given that for any function $V(r, \theta, \phi)$

$$\nabla^2 V = \frac{1}{r} \frac{\partial^2}{\partial r^2} (rV) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

10. It is sometimes said that the problems which wave mechanics studies are so over-simplified that they bear little relationship to experimental physics. Use your knowledge of the applications which are made of wave mechanics to comment on this statement.